

18. Convective equilibrium:

When all the parts of a fluid are interchanged and not sensibly influenced by radiation and conduction, the temperature of the fluid is said to be in a state of convective equilibrium.

This change is purely adiabatic.

$$\therefore PV^Y = \text{const.}$$

$$\frac{m}{V} = p \Rightarrow \frac{m}{p} = V.$$

$$\Rightarrow P \cdot \left(\frac{m}{p}\right)^Y = \text{const.}$$

$$\Rightarrow P = K p^Y]$$

K is constant.

18. A gaseous atmosphere is in equilibrium (or convective equilibrium), in such a way that

$$P = K p^Y = R p T \quad \dots \dots \quad (1)$$

where P , p and T are respectively the pressure, density and temperature (absolute) and K, Y, R are constants. If H be height of homogeneous atmosphere, and T_0 the temperature at sea-level where $z=0$, show that

$$\frac{T}{T_0} = 1 - \frac{Y-1}{Y} \frac{z}{H}.$$

Soln The pressure at a height z is given by

$$dp = -gpdz \quad \dots \quad (2)$$

Where p and ρ denote pressure and density at a height z .

Also from $P = K p^Y$, we have

$$dp = +K Y p^{Y-1} dP$$

$$\therefore (2) \Rightarrow +K Y p^{Y-1} dp = -g p dz$$

$$\Rightarrow K Y p^{Y-2} dp = -g dz$$

Integration

$$\frac{K\gamma}{\gamma-1} p^{\frac{\gamma}{\gamma-1}} = C - g \frac{s}{R} \Rightarrow \frac{p}{\gamma-1} \frac{p}{p} = C - g \frac{s}{R} \text{ as } p = kp^{\gamma}$$

or $\frac{\gamma}{\gamma-1} RT = C - g s \quad \text{as } p = RPT$
 $\Rightarrow \frac{P}{P} = RT$

$$\Rightarrow \frac{dT}{ds} = - \frac{g}{R} \cdot \frac{\gamma-1}{\gamma}$$

If T_0 be the temperature when $s=0$, then

$$\frac{\gamma}{\gamma-1} RT_0 = C - g \cdot 0$$

$$\text{or } \frac{\gamma}{\gamma-1} RT = \frac{\gamma}{\gamma-1} RT_0 - gs \dots$$

$$\Rightarrow \frac{\gamma}{\gamma-1} R(T - T_0) = -gs \dots$$

$$\Rightarrow \frac{\gamma}{\gamma-1} R \left(\frac{T}{T_0} - 1 \right) = - \frac{gs}{T_0}$$

$$\Rightarrow \frac{T}{T_0} - 1 = - \frac{\gamma-1}{\gamma} \cdot \frac{gs}{RT_0}$$

$$\Rightarrow \frac{T}{T_0} = 1 - \frac{\gamma-1}{\gamma} \frac{gs}{RT_0} \quad \text{--- (3)}$$

now, H denotes the height of the homogeneous atmosphere.

$$R \rho_0 T_0 = P_0 = g \rho_0 H$$

$P_0 = P_0$, $P = P_0$
 where air pressure
 and density on
 earth's surface
 is $s=0$.

$$\therefore RT_0 = gH$$

$$\therefore T_0 = \frac{gH}{R} \quad \text{--- (4)}$$

$$\therefore (3) \& (4) \Rightarrow \frac{T}{T_0} = 1 - \frac{\gamma-1}{\gamma} \frac{gs}{gH}$$

$$\Rightarrow \frac{T}{T_0} = 1 - \frac{\gamma-1}{\gamma} \frac{s}{H} \quad \text{--- (4)} \quad \begin{matrix} \rightarrow 2018 \\ \text{for question} \end{matrix}$$

isothermal process: In an isothermal process, heat can be added or released from the system just to keep the same temperature.

(notes) While in an adiabatic process, there is no heat added or released.
 or Isothermal process is a change of a system, in which the temperature remains constant. (i.e., $\Delta T = 0$).

Ex
107 [First article]
118 The height of the Torricellian vacuum in a barometer is a inches and the instrument indicates a pressure of b inches of mercury when the true reading is c inches. If the faulty readings are due to an imperfect vacuum, prove that the true reading corresponding to an apparent reading of d inches is $d + \frac{a(c-b)}{a+b-d}$.

Soln Let A be the area of cross section of the tube. The volume of air in the Torricellian vacuum is Aa cu.inches.

$\therefore c$ is the ~~true~~ height, the faulty reading b is due to depression in the mercury on account of the air in the tube.
 \therefore the pressure exerted by the air = $(c-b)$ inches of mercury.

Now, let x inches be the true reading when the apparent reading is d inches. Clearly, now the pressure exerted by the air in the Torricellian vacuum is equal to $(x-d)$ inches of mercury.

Now, the volume of air in the vacuum

= (total volume of the barometer from the top to the surface of mercury) - (the volume corresponding to d inches of mercury).

$$= \{(a+b)-d\} A$$

\therefore By Boyle's Law,

$$\begin{aligned} P_1 V_1 &= P_2 V_2 \\ [(p_0(c-b))Aa] &= [(p_0(x-d))((a+b)-d)]A \quad [P \rightarrow \text{density of mercury}] \\ \Rightarrow (c-b)Aa &= (x-d)\{(a+b)-d\}A \end{aligned}$$

$$\Rightarrow x-d = \frac{a(c-b)}{a+b-d}$$

$$\Rightarrow x = d + \frac{a(c-b)}{a+b-d}$$

\checkmark If masses m and m' of two gases in which the ratio of the pressure to the density are respectively k and k' , are mixed at the same temperature. Prove that the ratio of the pressure to the density in the compound is

$$\frac{mk + m'k'}{m + m'}$$

Soln: Let p_1, v_1, ρ_1 and p_2, v_2, ρ_2 be the pressure, volumes and densities of two gases at the same temperature.

Let P be the pressure and V be the volume of the compound; then

$$PV = p_1v_1 + p_2v_2$$

$$\therefore P \left(\frac{m}{P_1} + \frac{m'}{P_2} \right) = p_1 \cdot \frac{m}{P_1} + p_2 \cdot \frac{m'}{P_2} \quad \dots (1)$$

$$\text{But } (1) \quad p_1 = kp_1 \text{ and } p_2 = k'p_2$$

$$\therefore (1) \Rightarrow P \left(\frac{m}{P_1} + \frac{m'}{P_2} \right) = k \cdot p_1 \cdot \frac{m}{P_1} + k'p_2 \cdot \frac{m'}{P_2}$$

$$\Rightarrow P = \frac{mk + m'k'}{m(P_1 + m'P_2)} \quad \dots (3)$$

Also the density of the compound,

$$\begin{aligned} \rho &= \frac{m_1p_1 + m_2p_2}{v_1 + v_2} \\ &= \frac{\frac{m}{P_1} \cdot p_1 + \frac{m'}{P_2} \cdot p_2}{\frac{m}{P_1} + \frac{m'}{P_2}} \\ &= \frac{m + m'}{mP_2 + m'P_1} \quad \dots (4) \end{aligned}$$

$$\therefore (3) \div (4) \Rightarrow \frac{P}{\rho} = \frac{(mk + m'k')P_1P_2}{(mP_2 + m'P_1)} \times \frac{mP_2 + m'P_1}{(m + m')P_1P_2}$$

$$= \frac{mk + m'k'}{m + m'}$$

proved.

Ex.
2017

If the law connecting the pressure and density of the air were $p = kp^n$, prove that neglecting variations of gravity and temperature, the height of the atmosphere would be $\frac{n}{n-1}$ times the homogeneous atmosphere.

Sol Let p be the pressure and ρ be the density at any point. Then as given

$$p = kp^n$$

$$\text{or } p^{\frac{1}{n}} = k^{\frac{1}{n}} p. \quad \dots \quad (1)$$

The pressure at a height z above the earth's surface is given by

$$dp = -g\rho dz \quad \dots \quad (2)$$

$$(2) \div (1) \Rightarrow \frac{dp}{p^{\frac{1}{n}}} = -\frac{g}{k^{\frac{1}{n}}} dz$$

Integrating,

$$\int p^{\frac{1}{n}} dp = -gk^{\frac{1}{n}} z + C$$

$$\Rightarrow \frac{p^{\frac{n+1}{n}}}{\frac{n+1}{n}} = C - gk^{\frac{1}{n}} z$$

$$\Rightarrow \frac{n}{n+1} \cdot p^{\frac{n-1}{n}} = C - gk^{\frac{1}{n}} z$$

$$\Rightarrow \frac{n}{n+1} \cdot p \cdot \left(\frac{k}{p}\right)^{\frac{1}{n}} = C' - gz \quad \begin{cases} \text{where } C' = \frac{C}{k^{\frac{1}{n}}} \\ = C' k^{\frac{1}{n}} \end{cases}$$

$$\therefore \frac{n}{n+1} p \cdot \frac{1}{p} = C' - gz. \quad \text{by (1)} \quad \boxed{\text{Dividing by } k^{\frac{1}{n}}.}$$

Let h_1 be the height of the heterogeneous atmosphere when $p=0$ i.e. $z=h_1$ when $p=0$.

Then we have

$$0 = C' - gh_1 \Rightarrow C' = gh_1$$

$$\therefore \frac{n}{n+1} \cdot \frac{p}{p_0} = g(h_1 - z)$$

on the earth's surface, $z=p_0$ and $p=p_0$ say,

$P = P_0$ and $g = g_0$ when $z = 0$.

$$\therefore \frac{n}{n-1} \cdot \frac{P_0}{P_0} = g(h_1 - 0)$$

$$\Rightarrow h_1 = \frac{n P_0}{(n-1) g P_0} \quad \rightarrow (3)$$

(3) gives the height of the heterogeneous atmosphere when the pressure on the earth's surface is P_0 .

Let h_2 be the height of the homogeneous atmosphere for a pressure P_0 at the surface of the earth.

$$\text{Then } P_0 = P_0 g h_2$$

$$\Rightarrow g P_0 = \frac{P_0}{h_2} \quad \rightarrow (4)$$

From (3) & (4),

$$h_1 = \frac{n P_0}{(n-1) \frac{P_0}{h_2}}$$

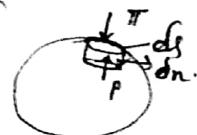
$$\Rightarrow h_1 = \frac{n}{n-1} \cdot h_2$$

which is the desired result. proved.

2017 Work done in compressing a gas:

Let p be the pressure and v the volume of the gas. Also let there be an external atmospheric pressure Π on the surface of the containing vessel.

Let us consider an elementary surface dS . Then pressure on this element = $(p-\Pi)dS$. If dn denote the inward normal displacement of this element dS , then the external work necessary to overcome the opposing pressure = $(p-\Pi)dS \cdot dn$.



$\therefore \delta A \cdot dn = \text{decrease in volume due to the displacement of the surface}$
 $= -\delta v$

$\therefore \text{Work required to reduce the volume by this amount} = -(p-\Pi) \delta v$

If the volume of the gas changes from V to V' , the work done

$$= - \int_V^{V'} (P - \pi) dV \quad \text{--- (1)}$$

Now there arise two cases:

(i) When the change is isothermal.

Then we have $PV = \text{const}$.

$$= C, \text{ say.}$$

$$\Rightarrow P = \frac{C}{V}.$$

$$\text{Then work} = - \int_V^{V'} \left(\frac{C}{V} - \pi \right) dV \quad \text{(by (1))}$$

$$= + C \log \frac{V}{V'} - \pi (V - V')$$

$$= + PV \log \frac{V}{V'} - \pi (V - V') \quad \left[\because PV = C \right]$$

where P denotes the pressure when volume is V .

(ii) When the change is adiabatic.

We have in this case $PV^\gamma = C$

$$\Rightarrow P = C V^{-\gamma}.$$

$$\therefore \text{work} = - \int_V^{V'} \left(C V^{-\gamma} - \pi \right) dV$$

$$= \frac{C}{\gamma-1} \cdot \left(\frac{1}{V'^{\gamma-1}} - \frac{1}{V^{\gamma-1}} \right) - \pi (V - V')$$

$$= \frac{PV'}{\gamma-1} \left(\frac{1}{V'^{\gamma-1}} - \frac{1}{V^{\gamma-1}} \right) - \pi (V - V')$$

$$= \frac{P'V' - PV}{\gamma-1} - \pi (V - V') \quad \left[\because PV = P'V' \right]$$

Note: If there is no atmospheric pressure, then the corresponding results are obtained by putting $\pi = 0$.

118

Q. Use of at Example of application of atmospheric pressure in daily life.

1. A vacuum cleaner has fan inside that creates a low pressure inside the device. Consequently, air and dirt particles are sucked into the device.
2. When air is sucked out of a drinking straw, the air pressure inside it decreases and the atmospheric pressure outside forces the liquid to go inside the straw.
3. Syringes are used to take blood for blood tests. The pressure of the liquid forces the liquid (blood) to move into the syringe when its plunger is withdrawn.